Milton Math Tournament
Junior Varsity Individual Written Test

PART I: Multiple Choice

1. Find the slant asymptote of $x^2y - x^3 - x^2 + y - x = 0$

a) y = 0
b) y = x
c) y = x + 1
d) y = 2x + 1
e) none of the above

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2. Find $|a + b|$ if a and b are negative integers such that $ab + a + b = 96$

a) 96
b) 45
c) 30
d) 98
e) none of the above

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3. $\sqrt{0.44444\ldots} =$

a) 0.202020...
b) 0.222222...
c) 0.666666...
d) 0.606060...
e) none of the above

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4. Let x and y be positive integers such that $x < y$ and $xy = 29(x + y)$. Find y/x.

a) 27
b) 28
c) 29
d) 30
e) none of the above

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5. Let r, s, and t be the roots to the equation $x^3 - 6x^2 - 27x + 140 = 0$. Find the sum

$$\frac{1}{rs} + \frac{1}{st} + \frac{1}{tr}$$

a) -3/70
b) 1/140
c) -1/2
d) -27/140
e) none of the above

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6. The base b numbers 104, 46, and 19 form a geometric sequence. Find the next term of this sequence in base b.

a) 6
b) 8
c) 10
d) 11
e) none of the above

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7. Define

$$f(x,n) = \sum_{k=1}^{n} \log_x \left(\frac{k}{x}\right)$$

Solve $f(x,10) = f(x,11)$ for x.

a) 9
b) 10
c) 11
d) 12
e) none of the above

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8. Let P and Q be polynomials such that $P(x)$ and $Q(P(Q(x)))$ have the same roots. If the degree of P is 7, then what is the degree of Q?

a) 0
b) 1
c) 2
d) 7
e) none of the above

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9. How many solutions does the equation $\sin x = 1/8$ have if $0^\circ \leq x < 180^\circ$?

a) 0
b) 1
c) 2
d) 3
e) none of the above

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10. A rhombus has a 60° angle as well as a side of length 2. What is the length of the longest diagonal?

a) 2
b) $2\sqrt{3}$
c) $3\sqrt{2}$
d) $\sqrt{3}$
e) none of the above

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11. The probability that event A occurs is p. The probability that event B (which is independent of A) occurs is $1/7$. Find p if $P(B) = P(\text{neither } A \text{ nor } B)$.

a) 0
b) $1/6$
c) $5/6$
d) $6/7$
e) none of the above

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12. Simplify

$$\frac{6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1}{(2x+1)(x^2+x+1)}$$

a) $3x^2 + x + 1$
b) $3x^2 + 2x + 1$
c) $3x^2 + 3x + 1$
d) $(x+1)(3x+1)$
e) none of the above

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13. The first term of an infinite geometric series is a, and the second one is b (where $|b| < |a|$). Find the sum of this series.

a) $a^2/(a-b)$
b) $a/(1-a/b)$
c) $a/(a^2-b^2)$
d) $a^3/(ab-b^2)$
e) none of the above

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14. Triangles $\triangle ABC$ and $\triangle DEF$ have sides of length a, b, c and d, e, f respectively. Which of the following can be used for proving congruence of $\triangle ABC$ and $\triangle DEF$? Assume $[\triangle ABC]$ represents the area of $\triangle ABC$.

I. $[\triangle ABC] = [\triangle DEF]$ II. $[\triangle ABC] = [\triangle DEF]$ and $a = d$

III. $[\triangle ABC] = [\triangle DEF]$, $a = d$ and $\angle A = \angle D$

a) II only
b) III only
c) II and III only
d) I, II, and III
e) none of the above

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15. The ratio of an arithmetic sequence's first term to its common difference is 7. What is the ratio of the seventh term to the sixth term?

a) 7
b) 13/12
c) 14/13
d) 12/11
e) none of the above

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16. If a is a rational number whereas b and c are irrational, which of the following can be rational?

I. $a+b$ II. $b+c$ III. abc

a) None
b) I only
c) III only
d) II and III only
e) none of the above

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17. How many points determine an ellipse?

a) 3
b) 4
c) 5
d) 6
e) none of the above

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18. Solve the equation

$$\log_6(x) + \frac{1}{\log_x(6)} = 3$$

a) 6
b) 36
c) $\sqrt{216}$
d) 216
e) none of the above

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19. In rectangle $ABCD$, $AB = 4$ and $BC = 7$. M is on CD such that $CM = 4MD$. Find the ratio of the area of $ABCD$ to that of triangle BCM.

a) 1/5
b) 2/5
c) 3/5
d) 4/5
e) none of the above

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20. The operation Δ is defined for all a and b as

$$a\Delta b = ab - b\Delta 2$$

For what a does $a\Delta 2 = 100$?

a) 49
b) 50
c) 51
d) There is more than one answer
e) none of the above

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21. A unit square has its lower left vertex at the origin. Two parallel lines, one passing through the origin, the other passing through the point $(0, b)$, cut the square into three pieces. If the area of the piece between the two lines is $5/6$ and one of the lines passes through $(1, 1)$, then find b.

a) 1/6
b) 1/3
c) 1/2
d) 5/6
e) none of the above

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22. Bob is standing on a straight road in the coordinate plane (with equation $y = 3x + 7$). What is the length of the shortest path he can take to the road $y = 3x + 19$?

a) 12
b) $6\sqrt{10}/5$
c) $12/\sqrt{2}$
d) 6
e) none of the above

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23. How many primes exist between $20!$ and $20! + 5$?

a) 0
b) 1
c) 2
d) 3
e) 4

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24. Find the value of x that satisfies the equation

$$\log\left(\frac{x^{1/x}}{x^{1/(x+1)}}\right) = \frac{1}{5050}$$

a) 1
b) 10
c) 100
d) 1000
e) none of the above

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25. An ant and a piece of candy are at opposite corners of a square. The ant, in one move, chooses one of the two sides at random and moves along it to the next corner. Then he chooses randomly again (since he is not the most intelligent ant) and so on. What is the expected number of moves for the ant to reach the candy?

a) 2
b) 3
c) 4
d) 5
e) more than 5

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26. The sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ... is such that the integer n appears exactly n times. Find the 50th term of this sequence.

a) 9
b) 10
c) 11
d) 12
e) none of the above

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27. If $f(g(x)) = \log(x^2 - 3)$ and $f(x) = -\log(x - 4)$, then find $g(2)$.

a) 4
b) 5
c) 6
d) 7
e) none of the above

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28. If $\log_3 7 = a$ and $\log_3 8 = b$, then find, in terms of a and b, the value of $\log_3 686$.

a) $3(a + b)$
b) $a/3 + 3b$
c) $3a + b/3$
d) $(a + b)/3$
e) none of the above

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29. If L is the value of y when the lines $y = mx+b$ and $y = x/m+1/b$ meet, then find $bL(m^2-1)$.

a) $m(1 - b^2)$
b) $m^2 - b^2$
c) $b(1 - m^2)$
d) $b^2 - m^2$
e) none of the above

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30. The equation $(x-1)(x-2)(x-3) = 24$ has the real root a and the complex roots b and c. Find bc/a.

a) does not exist
b) 1
c) 5/6
d) 6/5
e) none of the above

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31. Caffeinated Joe is painting a fence. Since Joe relies completely on java to get himself moving, his rate of work is variable: he works at his normal (constant) rate for 3 hours, works at half his rate for an hour, and then twice his normal rate for an hour, after which the cycle repeats itself. In terms of his normal rate R, what is Joe's average rate of work?

a) R
b) $4R/3$
c) $3R/2$
d) $2R$
e) none of the above

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32. What do you get when you round the number

$$\frac{1}{2^{2/3}-1} - \frac{1}{2^{2/3}+1}$$

up to the nearest integer?

a) 0
b) 1
c) 2
d) 3
e) none of the above

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33. Find how many integral values $f(x)$ can take, where

$$f(x) = \frac{5}{1+31e^x}$$

a) 0
b) 1
c) 4
d) 5
e) none of the above

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34. Find the value of K, where

$$\log\log 4 + \log\log 25 = \log K + \log\log 2 + \log\log 5$$

a) 1
b) 2
c) 4
d) 10
e) none of the above

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35. Rectangle $ABCD$ has points E on AB and F on CD. Point G is chosen on BC so that the area of $\triangle EBG$ is the same as that of $\triangle FCG$. If $EB = 12$, $FC = 20$, and $BC = 16$, find CG.

a) 4
b) 6
c) 10
d) 12
e) none of the above

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1. During a science fair project on which you wish to slack off, you take two observations from a population; the data set you collected is {1,6}. Then, in order to falsify results, you make up the data set {1,1,1,1,1,1,1,1,6,6,6,6,6,6,6,6}. Find the standard deviation of the 'fake' sample.

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2. Find the minimum value of the function $f(x)$ for $x > 2$, where

$$f(x) = \lfloor x/3 \rfloor + x|x - 4|$$

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3. I am standing behind a 10-feet-tall lamppost so that I can completely avoid the sunlight. I can either stand a maximum distance of $x/2$ away from the post, or I can lay down on the ground with my head a maximum distance of x away from the post (my feet extend away from the post). If I'm 6 feet tall, find x.

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4. If $x, y, z > 1$, then find the minimum value of

$$\log_y x^2 + \log_z y^2 + \log_x z^2$$

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5. A box contains 2 black balls and 1 white ball. We remove balls from the box, one at a time, without replacement. Before each draw, we make a guess at the color to be chosen, with the most likely color our guess (when one is more likely). What is the expected number of correct guesses?

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6. The equation

$$\ln\left(\frac{k^{1/k}}{(k+1)^{1/(k+1)}}\right) = F(k) \cdot \left[\ln\left(1 - \frac{1}{k+1}\right) + \frac{1}{k}\ln k\right]$$

is true for all k. Find $F(100)$.

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7. Given that $2x + y = 5$ and $x + 2y = 295$, find $x + y$.

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8. When expanded, find how many terms are in the sum

$$\sum_{k=1}^{5}\sum_{j=1}^{k} jk$$

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9. Bob sells carpets that are 6 feet in length. He calculated that, on a given day, his profits would equal 30% of his costs. However, when this 30% turned out to be only 20%, Bob realized that his yardstick was too long! How much longer than a real yardstick is Bob's yardstick?

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10. A triangle has sides a, b, and c satisfying $a < b < c$ and $\sqrt{a} + \sqrt{b} = \sqrt{c}$. What kind of triangle is it?

END OF WRITTEN TEST