JV Team Ciphering Solutions

1) **(80)** We need $5x + 2 = 67n$, for some integer n. Since $n = 1$ gives $x = 13$, we need $n > 1$. The first n that gives an integer x is $n = 6$, so that $x = 80$.

2) **(7/15)** Multiply through the first equation by $\sqrt{y}$ and let $u = \sqrt{x/y}$. Then the first equation becomes

$$\frac{8}{u} - 3 = \frac{5}{u+1} \Rightarrow u^2 = 8/3$$

so that $x = 8y/3$. Substitute that into the second equation to get $y = 16y - 7 \Rightarrow y = 7/15$.

3) **($\sqrt{5}$)** Let the sides be a, ar, ar^2. Then the triangle inequality demands that $a + ar > ar^2 \Rightarrow r < \frac{\sqrt{5}+1}{2}$. The inequality also states that $ar^2 + ar > a \Rightarrow r > \frac{\sqrt{5}-1}{2}$. Combining these gives

$$\frac{\sqrt{5}-1}{2} < r < \frac{\sqrt{5}+1}{2}$$

so that $a + b = \sqrt{5}$.

4) **(3)** This is an exercise in the process of elimination. Immediately throw out $n = 1, 2, 5, 10$ as they give no money at the end of the month. Throw out 9, as it gives only \$1 at the end. Choosing 8 will only give Gina \$1536. Throw it out. For the same reason, throw out 7. (At this point, it should be obvious that the answer will be small–either 3 or 4.) In fact, Gina should choose $n = 3$ and not 4, since $7^10 > 6^10 + 6^9$.‘

5) **(12)** There are three cases: The exponent is 0 (but the base is non-zero); the base is 1; or the base is -1 when the exponent is even. The viable solutions are $x = 5 - \sqrt{2}, 5 + \sqrt{2}, -3, 5$. Their sum is 12.

6) **(36)** Simply plug in to get $6\nabla 2 = 8$, and $8\nabla 2 = 36$.

7) **($3 - \sqrt{3}$)** Spatial reasoning required. To find r, we consider the diagonal of the cube in two ways. First, it has length $4\sqrt{3}$. However, it also has length $2r\sqrt{3}+2r$, as a sketch of the situation would show. Equating these gives $r = 3 - \sqrt{3}$.

8) **(90)** Because of the reflective property of the walls, they can be ignored! Imagine a typical parabola stretching for 200 feet. Let d be the cannon's distance from the wall as it was before. Basically, we know that the parabola passes through the points $(0, 0), (200, 0), (d, 50), (d + 100, 80)$. Substituting these into the general parabola equation $y = ax^2 + bx + c$ and solving for (a, b, c, d) yields $(a, b, c, d) = (-9/1000, 9/5, 0, 100/3)$. The maximum height occurs at $x = -b/2a = 100$ so that $y = 90$.

9) **(6)** Cube the equation and let $n = 3x$. This gives $n^n = 2^{2001} = 256^{250.125}$. Thus n is approximately equal to 256, and x is approximately equal to 85.33. Since x lies between 64 and 128, $\lfloor \log_2 x \rfloor = \log_2 64 = 6$.

10) **(5/9)** Let p be the probability that I don't sneeze at all during a day. Then $p^7 = 1 - 2059/2187 = 128/2187 \Rightarrow p = 2/3$. The probability that I don't sneeze during the next two days is $p^2 = 4/9$, so that the desired probability is $1 - p^2 = 5/9$.