JV Written Test Solutions

1) **(C)** Solve for y and long divide to get

$$y = x + 1 - \frac{1}{x^2 + 1}$$

So the asymptote is $y = x + 1$

2) **(E - 100)** Trial and error yields $(a, b) = (-2, -98)$, so that $|a + b| = 100$.

3) **(C)** $\sqrt{0.444\ldots} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666\ldots$

4) **(C)** Let $y = 29n$. Then, the equation becomes $xn = x + 29n$. Solve for x to get

$$x = \frac{29n}{n-1} = 29 + \frac{29}{n-1}$$

For x to be an integer, $\frac{29}{n-1}$ must also be an integer. This yields $n = 30$, so that the solution to the original equation is $(x, y) = (30, 29 \cdot 30)$, so that $y/x = 29$.

5) **(A)** The roots are 7, 4, and -5. Then the sum is -3/70

6) **(B)** In terms of b, the base ten representation of the numbers are $b^2 + 4$, $4b + 6$, and $b + 9$. For them to be a geometric sequence,

$$\frac{b^2 + 4}{4b + 6} = \frac{4b + 6}{b + 9} \Rightarrow b = 11$$

The sequence is 125, 50, 20; the next term is 8.

7) **(C)** Since $f(x, 11) = \log_x \left(\frac{11}{x}\right) + f(x, 10)$, the equation becomes

$$0 = \log_x \left(\frac{11}{x}\right) \Rightarrow x = 11$$

8) **(B)** Let n be the degree of Q. Then $Q(P(Q))$ has degree $n \cdot 7 \cdot n = 7n^2$. If $7n^2 = 7$, then $n = 1$.

9) **(C)** The angle x can be acute or obtuse. Recall that supplementary angles have the same value of sine. Thus there are exactly 2 solutions.

10) **(B)** This rhombus is composed of two equilateral triangles. Use a coordinate plane with the distance formula.

11) **(C)** The equation becomes, after substitutions, $1/7 = (6/7)(1 - p) \Rightarrow p = 5/6$.

12) **(C)** Divide to get $3x^2 + 3x + 1$.

13) **(A)** The ratio of the sequence is b/a, so that the sum is

$$\frac{a}{1-r} = \frac{a}{1-b/a} = \frac{a^2}{a-b}$$

14) **(E - Neither I,II, nor III)** To debunk III solves the problem, so consider the case where $\triangle DEF$ is $\triangle ABC$ reflected about the perpendicular bisector through BC. The conditions are satisfied, but the triangles are not congruent.

15) **(B)** We are given $a/d = 7$. This means the ratio r is given by

$$r = \frac{a_7}{a_6} = \frac{a+6d}{a+5d} = \frac{a/d+6}{a/d+5} = \frac{13}{12}$$

16) **(D)** I. does not work since a finite number of decimal places (a) cannot impact an infinite number of decimal places (b). II works, since $(\sqrt{2})+(2-\sqrt{2}) = 2$. III works since $(1)(\sqrt{2})(\sqrt{2}) = 2$.

17) **(C)** The general form for an ellipse is $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Dividing this by A gives $x^2 + Bxy/A + Cy^2/A + Dx/A + Ey/A + F/A = 0$. It takes 5 points to uniquely determine the coefficients B/A, and so on. Note that for a circle, the equation reduces to $x^2 + y^2 + Ax + By + C = 0$ (no xy term is needed). Thus, for a circle, only three points are needed (a well-known fact).

18) **(C)** Use $\log_a b = 1/\log_b a$ and other log properties to find $x = 6^{3/2} = \sqrt{216}$.

19) **(E - 5/2)** BCM has legs of length $16/5$ and 7. Its area is $56/5$. The rectangle's area is 28. The ratio is $5/2$.

20) **(C)** Let $a = b = 2$ to find that $2\Delta 2 = 2$. Then $a\Delta 2 = 2a - 2$. Solving $2a - 2 = 100$ yields $a = 51$.

21) **(D)** The top chunk of the square is a right triangle with legs of length $1-b$ and 1. The area is $1/12$. Thus $b = 5/6$.

22) **(B)** The distance between the lines $y = mx + b$ and $y = mx + c$ is

$$\frac{|b-c|}{\sqrt{m^2+1}}$$

So the distance is $6\sqrt{10}/5$.

23) **(E)**

24) **(C)** Rearrange.

$$\frac{1}{5050} = \frac{1}{x(x+1)} \log x \Rightarrow x = 100$$

25) **(C)**

26) **(B)**

27) **(B)**

28) **(C)**

29) **(B)** Solve the system.

30) **(D)** 5 is the real root, and so the equation becomes $(x-5)(x^2-x+6)=0$. $bc/a = 6/5$.

31) **(E - 11R/10)** In 5 hours, Joe works at an average rate of

$$\frac{1}{5}(3R + R/2 + 2R) = 11R/10$$

32) **(C)** Rearrange to get

$$\frac{2}{16^{1/3}-1}$$

Since

$$1 = \frac{2}{27^{1/3}-1} < \frac{2}{16^{1/3}-1} < \frac{2}{8^{1/3}-1} = 2$$

when rounded up, the number is 2.

33) **(C)** The range of the function is $(5/31, 5)$. There are 4 integers in that range.

34) **(C)**

$$\log\log 4 + \log\log 25 = \log((\log 4)\cdot(\log 25)) = \log(4\cdot(\log 2)\cdot(\log 5)) = \log 4 + \log\log 2 + \log\log 5$$

So $K = 4$.

35) **(B)** Let $CG = x$. Then, $(20)(x)/2 = (12)(16-x)/2 \Rightarrow x = 6$.

PART II : Free Response

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1) **($2\sqrt{15}/3$)** The formula for standard deviation is, for sample size n and mean μ,

$$\sigma = \sqrt{\frac{1}{n-1}\sum(x_i - \mu_i)^2}$$

So that in this case, $\sigma = 2\sqrt{15}/3$

2) **(1)** The value of $\lfloor x/3 \rfloor$ only gets larger, so focus on $x|x-4|$. When $2 < x < 4$, it becomes $4x - x^2$, which is decreasing. When $x \geq 4$, it becomes $x^2 - 4x$, which is increasing. The minimum must then occur at $x = 4$. $f(4) = 1$.

3) **(24)** Draw a picture; use similar triangles. You get

$$\frac{10}{x+6} = \frac{6}{x/2+6} \Rightarrow x = 24$$

4) **(6)** By symmetry, let $x = y = z$ to get the expression equals 6.

5) **(7/3)** Consider cases: BBW, BWB, WBB. In the first case, we guess black first, make a random guess second, and guess correctly last. That's 2.5 correct guesses. In the second case, we guess correctly first, make a random guess second, and guess correctly last. That's also 2.5 correct guesses. In the last case, we guess wrong first and correctly both times after. That's 2 correct guesses. The total correct guesses is their average: $(2.5 + 2.5 + 2)/3 = 7/3$.

6) **(1/101)** Rearrange. $F(k) = 1/(k+1)$.

7) **(100)** Add the equations and divide by 3 to get $x + y = 100$.

8) **(15)** When $k = 1$, there is one term. Then $k = 2$ adds two more terms, just as $k = 3$ adds 3, etc. The total number of terms is $1 + 2 + 3 + 4 + 5 = 15$.

9) **(1.5 feet)** Let the cost be C and the extra length be x. We can assume that the cost of a carpet is proportional to the length, so that, for some constant k, $C_1 = 6k$ is what Bob expected. What actually happened is $C_2 = (6 + 2x)k$ (since Bob uses the yardstick twice to measure a carpet). These two values of C are related as $.3C_1 = .2C_2 \Rightarrow 1.8k = (1.2 + .4x)k \Rightarrow x = 1.5$.

10) **(obtuse)** Square the equation to get $a + b + 2\sqrt{ab} = c$. It is already clear that $c^2 > a^2 + b^2$, so the triangle is obtuse.