Milton Math Tournament
Junior Varsity Team Ciphering

1. Find the smallest integer x greater than 13 such that the following fraction is reducible:

$$\frac{67}{5x+2}$$

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2. Find the value of y, given that

$$\frac{8}{\sqrt{x}} - \frac{3}{\sqrt{y}} = \frac{5}{\sqrt{x}+\sqrt{y}} \qquad \text{and} \qquad y = 6x - 7$$

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3. A triangle has sides whose lengths form a geometric sequence. For this to occur, the common ratio r of the sequence must satisfy $a < r < b$. Find $a + b$.

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4. Greedy Gina has a rich uncle. Gina's uncle knows of his niece's greed and decides to test her mettle. Thus he makes this proposition:

> "Gina, as you know, this is day 0 of a month that goes to day 10. You choose an integer n from 1 to 10, inclusive. I will take that number and pay you $(10-n)^t$ each day, where t is the current day (0, 1, 2, etc. up to 10). But there is a catch! Everyday that is a multiple of n, I will take back all the money that you currently have!"

What n should Gina choose to maximize her money at the end of the month (after day 10)?

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5. Find the sum of all x such that

$$(x^2 - 10x + 24)^{x^2-x-12} = 1$$

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6. Define $x \nabla y$ as

$$x \nabla y = x^2 - \binom{y^3}{2}$$

What is the value of $(6 \nabla 2) \nabla 2$?

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7. What is the largest possible value of r such that two balls of radius r can be placed completely within a cube of side length 4?

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8. A room with an infinitely high ceiling is 100 feet long, wall to wall. I shoot a toy cannon at the wall from the ground. The cannonball, while traveling in its parabolic arc, hits the wall 50 feet up and bounces off (obeying "angle of incidence equals angle of reflection"). The ball hits the other wall 80 feet above the floor and bounces off, after which the ball hits my cannon! How high was the ball at its highest point?

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9. Given that $(3x)^x = 2^{667}$, find the value of $\lfloor \log_2 x \rfloor$, where $\lfloor n \rfloor$ represents the greatest integer function.

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10. While recovering from a cold during this last week, I determined that the probability that I sneeze in a given day is a constant. If the probability that that I sneeze at some point during the week is 2059/2187, what is the probability that I sneeze at some point during the next two days?