1. Definition Of A Square Number

A square number, also called a perfect square, is a number of the form n^2, where n is an integer. The square numbers for $n = 0, 1, 2, 3, 4, 5, 6$, and 7 are 0, 1, 4, 9, 16, 25, 36, and 49, respectively.

Property 1: Any even square number can be written as $(2n)^2 = 4n$, which is a multiple of 4.
Any odd square number can be written as $(2n + 1)^2 = 4n^2 + 4n + 1$, which is a multiple of 4 plus 1.

Property 2: There are no square numbers between two consecutive square numbers.

if $n^2 < a < (n + 1)^2$, then a is not a square number.

Problem 1: Find the smallest value of $a + b$ for any positive integers a and b such that $56a + 392b$ is a square number.
A. 6 B. 7 C. 8 D. 9

2) How many two-digit positive integers are there such that the sum of the two-digit positive integers and the number formed by reversing the digits of the two-digit positive integers is a square number?

3) (AMC) Let x and y be two-digit integers such that y is obtained by reversing the digits of x. The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m. What is $x + y + m$?

(A) 88 (B) 112 (C) 116 (D) 144 (E) 154

4) How many positive integers x are there such that both x and $x + 99$ are perfect squares?

(A) 1 (B) 2 (C) 3 (D) 49 (E) 99

Miscellaneous

5) If $pq = 21$, $qr = 132$, and $rp = 77$, and $p > 0$, then $p =$

(A) $\frac{49}{4}$ (B) $\frac{4}{49}$ (C) $\frac{11}{4}$ (D) $\frac{2}{7}$ (E) $\frac{7}{2}$

6) Chris can paint a house in 18 hours. He has been contracted to paint 5 houses. After painting 23 hours, Tom helps him and they finish in 40 hours. How long would it have taken Tom to paint 2 of the houses alone?

(A) 46 hours (B) 48 hours (C) 51 hours, $6\frac{2}{3}$ minutes
(D) 51 hours, 40 minutes (E) 53 hours, 20 minutes

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Problem 1: Find the smallest value of $a + b$ for any positive integers a and b such that $56a + 392b$ is a square number.

A. 6 B. 7 C. 8 D. 9

$56a+392b = 56(a+7b) = 8 \cdot 7(a+7b) = 4 \cdot 2 \cdot 7(a+7b)$

$4 \cdot 2 \cdot 7(14) = 4 \cdot 2^2 \cdot 7^2 =$ square number

7 1

$a=7$
$b=1$

2) How many two-digit positive integers are there such that the sum of the two-digit positive integers and the number formed by reversing the digits of the two-digit positive integers is a square number?

$10a+b+10b+a = m^2$

$11a+11b = m^2$

$11(a+b) = m^2$

$a+b=11$

$(a,b) = (2,9), (3,8), (4,7), (5,6), (6,5), (7,4), (8,3), (9,2)$

8 integers: 29, 38, 47, 56, 65, 74, 83, 92

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b^2 a^2

$a^2 - b^2 = 99$

$(a-b)(a+b) = 99$

	$(a-b)(a+b)$
$1 \cdot 99$	$(50-49)(50+49)$
$3 \cdot 33$	$(18-15)(18+15)$
$9 \cdot 11$	$(10-1)(10+1)$

3 integers:

$x = 1^2, 15^2, 49^2$

Miscellaneous

5) If $pq = 21$, $qr = 132$, and $rp = 77$, and $p > 0$, then $p =$

(A) $\frac{49}{4}$ (B) $\frac{4}{49}$ (C) $\frac{11}{4}$ (D) $\frac{2}{7}$ (E) $\frac{7}{2}$

$rp = 77$

$qrp = 77q$

$132p = 77q$

$q = \frac{21}{p}$

$132p = 77\left(\frac{21}{p}\right)$

$\frac{132p}{1} = \frac{77 \cdot 21}{p}$

$132p^2 = 77 \cdot 21$

$p^2 = \frac{77 \cdot 21}{132}$

$p^2 = \frac{7 \cdot 11 \cdot 3 \cdot 7}{4 \cdot 33}$

$p^2 = \frac{49}{4}$

$p = \sqrt{\frac{49}{4}}$

$p = 7/2$

6) Chris can paint a house in 18 hours. He has been contracted to paint 5 houses. After painting 23 hours, Tom helps him and they finish in 40 hours. How long would it have taken Tom to paint 2 of the houses alone? Total: 5 houses $= \frac{90}{18}$ houses

(A) 46 hours (B) 48 hours (C) 51 hours, $6\frac{2}{3}$ minutes
(D) 51 hours, 40 minutes (E) 53 hours, 20 minutes

Chris → $\frac{23}{18}$ houses finished, $\frac{90}{18} - \frac{23}{18} = \frac{67}{18}$ houses remaining

(hours)(houses/hr) = remaining houses

$(40)\left[\frac{1}{18} + \frac{1}{T}\right] = \frac{67}{18}$

$\frac{40}{18} + \frac{40}{T} = \frac{67}{18}$

$\frac{40}{T} = \frac{67}{18} - \frac{40}{18}$

$\frac{40}{T} = \frac{27}{18}$

$\frac{40}{T} = \frac{3}{2}$

$3T = 80$

$T = 80/3$

$2T = 2\left(\frac{80}{3}\right) = \frac{160}{3}$

$\frac{160}{3} =$ 53⅓ hrs.